KNOSOS: a fast orbit-averaged neoclassical code for arbitrary stellarator geometry

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Kinetic Theory Working Group Meeting 2018, Madrid

Aknowledgements: A. Mollén, C. Nuehrenberg (IPP), S. Satake, S. Morita, XL Huang (NIFS)
Motivation and goal

In stellarators, neoclassical transport:

- Tends to dominate radial particle and energy fluxes in the core of reactor-relevant plasmas [Dinklage, NF (2013)].

- Explains impurity accumulation observed in many stellarator plasmas [Burhenn, NF (2009)].
Goal of this work: **write a neoclassical code** (i.e., a code that solves the Drift Kinetic Equation together with the quasineutrality equation) **that is, at the same time, accurate and fast:**

- **Accurate so that:**
  - it gets the simulation quantitatively closer to the experiment;

- **Fast so that:**
  - it can be included in an optimization loop;
  - it allows to perform comprehensive parameter scans;

i.e., generally speaking, so that it improves confidence in our predictions and that these predictive capabilities can be fully exploited.
Energy transport simulations (short overview).

Equations.

Algorithms.

First simulations for arbitrary geometry.

Examples of other applications.
At low collisionality, stellarator neoclassical transport is very different from (and larger than in) tokamaks [Helander, PPCF (2012)].

Several strategies for characterizing magnetic configurations w.r.t. neoclassical energy transport:

1. **Calculation of the level of $1/\nu$ transport.**
   - One scalar, effective ripple, gives level of $1/\nu$ transport.
   - Fast calculation of $\varepsilon_{eff}$ provided by NEO [Nemov, PoP (1999)].
   - Fails to explain configuration dependence of the experimental $\tau_E$ stellarator scaling [Yamada, NF (2005)].
Calculation of $\tau_E$ solving the energy balance for every species $b$:

$$\frac{3}{2} \left< \frac{\partial n_b T_b}{\partial t} \right> + \frac{1}{V'} \frac{\partial}{\partial r} V' \left< Q_b \right> = \left< P_b \right>.$$ 

- $Q_b$ calculated with DKES [Hirshman, PoF (1986)]\(^1\).
- $\tau_E \sim \frac{\int dV \sum n_b T_b}{\int dV \sum P_b}$, gives level of transport for given configuration and sources [Turkin, PoP (2011), Geiger, PPCF (2015)].
- Monoenergetic approach by DKES (and others [Beidler, NF (2011)]) saves some time by calculating a database of transport coefficients instead of solving the DKE multiple times.
  - Still, for each new magnetic configuration, creating database takes time ($\sim 1 - 2$ days [Beidler, W7-X Workshop (2016)]). See also [Landreman, APS (2017)].
  - It oversimplifies the DKE (next slides).

\(^1\)The net energy source is provided by a suite of codes, and there is an ad-hoc anomalous model at the edge.
Accurate calculation of $Q_b$ for fixed kinetic profiles.

Recently, there have been refinements of some aspects of neoclassical theory that have lead to more complete neoclassical codes (EUTERPE, SFINCS, FORTEC-3D...):

- electric field tangent to magnetic surface (aka $\varphi_1$);
- magnetic drift tangent to magnetic surface;
- radially global effects;
- complete collision operator.

Calculations with all the ingredients can get extremely time-consuming and memory-demanding, specially at low collisionalities.

$\Rightarrow$ Comprehensive parameter scans may become impossible.
We have outlined three strategies for characterizing the energy transport of a magnetic configuration. Our goal is to create a code that can contribute to the three strategies by:

1. Giving figures of merit for neoclassical transport of collisionality regimes other than the $1/\nu$ (i.e. $\sqrt{\nu}$ and sb-p).
2. Employing a different "database of coefficients" that allows
   - to be very fast at low collisionalities and, at the same time,
   - to retain all the necessary ingredients of neoclassical theory.

KNOSOS solves the bounce-averaged drift kinetic and quasineutrality equations discussed in [Calvo, PPCF (2017)] for arbitrary geometry.
Coordinates on phase space

We are interested in calculating the ion\(^2\) distribution function

\[ F(\psi(x), \alpha(x), l(x), E, \mu, \sigma), \]

where:

- \( \psi \) is the radial coordinate.
  - We choose \( \psi = \psi_t \), where \( 2\pi \psi_t \) is the toroidal magnetic flux.
- \( \alpha \) is an angle that labels magnetic field lines on the surface.
  - We choose \( \alpha \equiv \theta - \iota \zeta \), where \( \theta \) and \( \zeta \) are Boozer angles and \( \iota \) is the rotational transform.
- \( l \) is the arc length along the magnetic field line.

\[
B = \nabla \psi \times \nabla \alpha,
\]

\[
b = \frac{B}{|B|} = \frac{B}{|B|}.
\]

- \( E = v^2 + Z e / m \varphi \) is the total energy per mass unit, \( v = |v| \), and \( \varphi(\psi, \alpha, l) \) is the electrostatic potential.
- \( \mu \) is the magnetic moment.
- \( \sigma = v_\parallel / v \) is the sign of the parallel velocity, \( v_\parallel = v \cdot b \).

\(^2\)We will take a mass ratio expansion \( \sqrt{m_e / m} \ll 1 \); all quantities will refer to ions and \( Z e, m \), etc, will have the usual meaning.
In these coordinates, the equation for the ion distribution function $F$ can be written as

$$(v_{\parallel} b + v_d) \cdot \nabla F = C[F, F],$$

where $v_d = v_M + v_E$ is the sum of magnetic and $E \times B$ drifts, and $C[F, F]$ is the (ion-ion) collision operator.

We know that the motion of particles along the magnetic field is much faster than the motion perpendicular to it.

At low collisionalities ($\nu_* \equiv \nu_{ii} R_0 / v_T \ll \epsilon^{3/2}$), the parallel streaming term also dominates over the collision term for both passing and trapped trajectories.

In an expansion in the normalized Larmor radius, the first term is:

$$v_{\parallel} \partial_l F = 0,$$

and, to next order, $F$ is determined by integrals over $l$ of the drift-kinetic equation.
Bounce-averaged drift kinetic equation

We have:

Passing particles:
\[ \int_0^{2\pi} \, d\alpha \int_0^{l_{\text{max}}(\psi,\alpha)} \frac{1}{|v_\parallel|} C[F,F] \, dl = 0. \]

Trapped particles:
\[ -\partial_\psi J \partial_\alpha F + \partial_\alpha J \partial_\psi F = \sum_\sigma \frac{Ze}{m} \int_{l_{b_1}}^{l_{b_2}} \frac{1}{|v_\parallel|} C[F,F] \, dl . \]

Everything has become expressed in terms of derivatives of the second adiabatic invariant:
\[ J(\psi, \alpha, \mathcal{E}, \mu) \equiv 2 \int_{l_{b_1}}^{l_{b_2}} |v_\parallel| \, dl . \]

At the bounce points, \( l_{b_1} \) and \( l_{b_2} \), \( v_\parallel = 0 \).
Bounce-averaged drift kinetic equation (II)

\[-\partial_\psi J \partial_\alpha F + \partial_\alpha J \partial_\psi F = \sum_\sigma \frac{Ze}{m} \int_{l_{b1}}^{l_{b2}} \frac{1}{|v_\parallel|} C[F, F] dl.\]

These derivatives of the second adiabatic invariant are connected to the drifts perpendicular to the magnetic field line:

\[\partial_\alpha J = \frac{2Ze}{m} \int_{l_{b1}}^{l_{b2}} \frac{1}{|v_\parallel|} v_d \cdot \nabla \psi \, dl, \quad \partial_\psi J = -\frac{2Ze}{m} \int_{l_{b1}}^{l_{b2}} \frac{1}{|v_\parallel|} v_d \cdot \nabla \alpha \, dl.\]

- \(\partial_\alpha J\) contains the radial magnetic and \(E \times B\) drift caused by the inhomogeneity of the magnetic field strength and of the electrostatic potential on the flux surface respectively.
- Can model \(1/\nu\), global effects and transport caused by \(\varphi_1\).
- \(\partial_\psi J\) contains the precession tangential to the flux surface caused by the radial variation of the magnetic field strength and of the electrostatic potential (i.e. \(E_r\)) respectively.
- Eqs. contain the \(\sqrt{\nu}\) and superbanana-plateau regimes.
We would like to solve a drift-kinetic equation that
- is linear and radially local and,
- rigorously includes the tangential magnetic drift.

It can be proven that [Calvo, PPCF (2017)] this can be done rigorously if the magnetic configuration is optimized enough. Then

\[ |\partial_\alpha J \partial_\psi g| \ll |\partial_\psi J \partial_\alpha g| , \]

where \( g(\psi, \alpha, v, \lambda) \) is the dominant piece of the non-adiabatic component of the deviation of the ion distribution function from a Maxwellian \( F_M \):

\[ g = F - F_M [1 - (Ze\varphi_1/T)] , \]

and we have changed coordinates to:

\[ v = 2\sqrt{\mathcal{E} - Ze\varphi_0/m} \quad \text{and} \quad \lambda = \mu/(\mathcal{E} - Ze\varphi_0/m) , \]

and used \( \varphi(\psi, \alpha, l) = \varphi_0(\psi) + \varphi_1(\psi, \alpha, l) \) with \( |\varphi_1| \ll |\varphi_0| \).

\(^3\)This also happens in large aspect-ratio stellarator for large values of the radial electric field.
The equations that determine $g(\psi, \alpha, v, \lambda)$ are:

$$\int_{l_{b1}}^{l_{b2}} \frac{dl}{|v||v|} v_d \cdot \nabla \alpha \partial_\alpha g + \int_{l_{b1}}^{l_{b2}} \frac{dl}{|v||v|} v_d \cdot \nabla \psi \Psi F_M = \int_{l_{b1}}^{l_{b2}} \frac{dl}{|v||v|} C^{lin}[g]$$

with $\int_0^{2\pi} g \, d\alpha = 0$ for trapped particles and $g = 0$ for passing.

and $\left(\frac{Z}{T} + \frac{1}{T_e}\right) \varphi_1 = \frac{2\pi}{en} \int_0^\infty dv \int_{B_{\text{max}}^{-1}}^{B^{-1}} d\lambda \frac{v^3B}{|v||v|} g$

- $\Psi = \frac{\partial_\psi n}{n} + \frac{\partial_\psi T}{T} \left(\frac{mv^2}{2T} - \frac{3}{2}\right) + \frac{Ze\partial_\psi \varphi_0}{T}$.

- $C^{lin}[g]$ is the linearized collision operator. We will use pitch-angle-scattering for the moment.

- $v_E \cdot \nabla \alpha \approx \partial_\psi \varphi_0$, so the system is linear in $\varphi_1$.

This is the set of equations solved by KNOSOS$^4$.

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$^4$ We do not use explicitly the expansion of [Calvo, PPCF (2017)] of the magnetic configuration around a perfectly optimized stellarator as in [Velasco, PPCF (2018)] but, for small $|\partial_\psi \varphi_0|$, if the stellarator is not optimized enough, it is not justified to use the local DKE.
Explicit expressions

Using the expressions of the drift in Boozer coordinates\(^5\),
everything gets written in terms of a few bounce integrals:

\[
\left( I_{v_M,\alpha}(\alpha, \lambda) + \frac{1}{v_d} I_{v_E,\alpha}(\alpha, \lambda) \right) \partial_\alpha g + \left( I_{v_M,\psi}(\alpha, \lambda) + \frac{1}{v_d} I_{v_E,\psi}(\alpha, \lambda) \right) F_M \Upsilon
\]

\[
= \frac{\nu \nu}{v_d} \partial_\lambda I_\nu(\alpha, \lambda) \partial_\lambda g,
\]

with \( v_d \equiv \frac{mv^2}{Ze} \) and

\[
I_{v_E,\alpha} = \partial_\psi \varphi_0 \int_{l_{b_1}}^{l_{b_2}} \frac{dl}{\sqrt{1 - \lambda B}}, \quad I_{v_M,\alpha} = \frac{1 - \lambda}{2} \int_{l_{b_1}}^{l_{b_2}} \frac{dl}{\sqrt{1 - \lambda B}} \frac{\partial_\psi B}{B},
\]

\[
I_{v_E,\psi} = -\int_{l_{b_1}}^{l_{b_2}} \frac{dl}{\sqrt{1 - \lambda B}} \partial_\theta \varphi_1, \quad I_{v_M,\alpha} = -\frac{1 - \lambda}{2} \int_{l_{b_1}}^{l_{b_2}} \frac{dl}{\sqrt{1 - \lambda B}} \frac{\partial_\theta B}{B},
\]

\[
I_\nu = \int_{l_{b_1}}^{l_{b_2}} dl \frac{\sqrt{1 - \lambda B}}{B}.
\]

\(^5\)For the sake of simplicity, in this talk we assume vacuum magnetic field, \( B = B_\zeta \nabla \zeta \).
Coefficients

Integrals in $l$ done using extended midpoint rule:
- Number of points tripled until the result converges.
- Note that integrals like:

$$I = \int_{l_1}^{l_2} dl \frac{f(l)}{\sqrt{1 - \lambda B(l)}}$$

diverge logarithmically close to the passing/trapped boundary. One can ease the convergence (and thus speed up the calculation) by removing the divergence and solving it analytically:

$$I = I_0 + \sum_j I_j,$$

$$I_0 = \int_{l_1}^{l_2} dl \left( \frac{f(l)}{\sqrt{1 - \lambda B(l)}} - \sum_j \frac{f(l_j)}{\sqrt{-\lambda(l - l_j)[\partial_l B|_{l_j} + \frac{1}{2} \partial^2_l B|_{l_j} (l - l_j)]}} \right),$$

$$I_j = \sqrt{-\frac{2}{\lambda \partial^2_l B|_{l_j}}} \left[ \ln \left( 2\lambda \sqrt{\frac{\partial^2_l B|_{l_j}}{-2}} \sqrt{-\partial_l B|_{l_j} x - \frac{1}{2} \partial^2_l B|_{l_j} x^2 - \lambda (\partial^2_l B|_{l_1} x + \partial_l B|_{l_1})} \right) \right]_{l_2 - l_1}^{l_1 - l_1}. $$
When integrating along the field line using fixed step \( \Delta \zeta, \Delta \theta \), instead of evaluating

\[
B(\theta, \zeta) = \sum_{m,n} B_{m,n} \cos[m\theta + n\zeta],
\]

\[
B(\theta + \Delta \theta, \zeta + \Delta \zeta) = \sum_{m,n} B_{m,n} \cos[m(\theta + \Delta \theta) + n(\zeta + \Delta \zeta)],
\]

\[
B(\theta + 2\Delta \theta, \zeta + 2\Delta \zeta) = \sum_{m,n} B_{m,n} \cos[m(\theta + 2\Delta \theta) + n(\zeta + 2\Delta \zeta)],
\]

we can precalculate a few sines and cosines \( \cos(m\theta + n\zeta) \), \( \sin(m\theta + n\zeta) \), \( \cos(m\Delta \theta + n\Delta \zeta) \) and \( \sin(m\Delta \theta + n\Delta \zeta) \) and use trigonometric identities to iterate:

\[
B(\theta + \Delta \theta, \zeta + \Delta \zeta) = \sum_{m,n} B_{m,n}[\cos(m\theta + n\zeta) \cos(m\Delta \theta + n\Delta \zeta) - \sin(m\theta + n\zeta) \sin(m\Delta \theta + n\Delta \zeta)]
\]
Discretization

We are left with two variables:

- Bounce-averages in \( l \).
- Pitch-angle scattering collision operator \( \Rightarrow v \) is a parameter.
  - Calculate \( \partial v Q_i \) for 28 values of \( v \) and integrate Gauss-Laguerre.
- Radially local equations \( \Rightarrow \psi \) is a parameter.

\[
\left( I_{vM,\alpha}(\alpha, \lambda) + \frac{1}{v_d} I_{vE,\alpha}(\alpha, \lambda) \right) \partial_\alpha g + \left( I_{vM,\psi}(\alpha, \lambda) + \frac{1}{v_d} I_{vE,\psi}(\alpha, \lambda) \right) F_M \Upsilon = \frac{v \nu}{v_d} \partial_\lambda I_v(\alpha, \lambda) \partial_\lambda g.
\]

Discretize \( g_{i,j} \equiv g(\alpha_i, \lambda_j) \), with uniform grid in \( \lambda \) and \( \alpha \).
Second order central differences.

- \( g = 0 \) at boundary between passing and trapped.
- \( v_\parallel \partial_\lambda g = 0 \) at well bottoms.

Subtlety: at bifurcations, \( g \) is continuous in \( \alpha \) and \( \lambda \), but \( \partial_\lambda g \) is not continuous in \( \lambda \).
\[
\begin{align*}
(I_{v_M,\alpha}(\alpha, \lambda) + \frac{1}{v_d} I_{v_E,\alpha}(\alpha, \lambda)) \partial_{\alpha} g + (I_{v_M,\psi}(\alpha, \lambda) + \frac{1}{v_d} I_{v_E,\psi}(\alpha, \lambda)) F_M \Upsilon = \frac{v\nu}{v_d} \partial_\lambda I_{\nu}(\alpha, \lambda) \partial_\lambda g,
\end{align*}
\]

**Arbitrary point:**
\[
\begin{align*}
\partial_{\alpha} g |_{i,j} &= \frac{g_{i+1,j} - g_{i-1,j}}{2\Delta \alpha}, & \partial_{\lambda} g |_{i,j} &= \frac{g_{i,j+1} - g_{i,j-1}}{2\Delta \lambda}, \\
\partial^2_{\lambda} g |_{i,j} &= \frac{g_{i,j+1} + g_{i,j-1} - 2g_{i,j}}{(\Delta \lambda)^2}.
\end{align*}
\]

**Bifurcations (\(W = I, II\ldots\)):**
\[
\begin{align*}
\partial_{\alpha} g |_{i,j} &= \frac{g_{i+1,j} - g_{i-1,j}}{2\Delta \alpha}, \\
\text{rhs} &= \sum_w I_{\nu,i,j+1,w} \frac{-g_{i,j+3,w} + 2g_{i,j+2,w} - 3g_{i,j+1,w}}{4(\Delta \lambda)^2} \\
&\quad - I_{\nu,i,j-1} \frac{g_{i,j} - g_{i,j-2}}{4(\Delta \lambda)^2}.
\end{align*}
\]

**Top:** \(g_{i,1} = 0\).

**Bottom:** \((j = N + 1): \) \(\text{rhs} = -I_{\nu,i,N-1} \frac{g_{i,N} - g_{i,N-2}}{4(\Delta \lambda)^2} \).
Linear problem

\[
\begin{aligned}
&\left( I_{vM,\alpha}(\alpha, \lambda) + \frac{1}{v_d} I_{vE,\alpha}(\alpha, \lambda) \right) \partial_{\alpha} \\
&+ \frac{v\nu}{v_d} \partial_{\lambda} \nu(\alpha, \lambda) \partial_{\lambda} \\
&g \\
\end{aligned}
\]

\[
\begin{bmatrix}
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots
\end{bmatrix}
\cdot
\begin{bmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{bmatrix}
=
\begin{bmatrix}
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{bmatrix}
\]

- Square matrix, \( N_\lambda \times N_\alpha \) elements per row.
  - Sparse, \( \sim 5 \) non-zero elements per row.
  - Non-zero elements are always at the same position for a given flux-surface, but relative weight varies with \( v, E_r \).
- Direct solver, based on LU factorization.
  - Will be useful for solving quasineutrality.
Benchmark with DKES \((v_M \cdot \nabla \alpha = 0, \varphi_1 = 0)\)

W7-X (standard configuration, EIM, low ripple) at:

- \(\psi / \psi_{LCMS} = 0.3\) (top right).
- \(\psi / \psi_{LCMS} = 0.5\) (bottom left).
- \(\psi / \psi_{LCMS} = 0.7\) (bottom right).
Benchmark with DKES ($\mathbf{v}_M \cdot \nabla \alpha = 0, \varphi_1 = 0$)

LHD ($R = 3.75$) at $\psi/\psi_{LCMS} = 0.5$ (top right).

QIPC at:
- $\psi/\psi_{LCMS} = 0.1$ (bottom left).
- $\psi/\psi_{LCMS} = 0.25$ (bott. right).
Benchmark with DKES \((v_M \cdot \nabla \alpha = 0, \varphi_1 = 0)\)

49 simulations at low collisionalities

- \(N_\alpha=32, \, N_\chi=256\).
- Total time: 2.4 s (0.05 s/point) at one single core of Dual Xeon quad-core 3.0 GHz (\(\sim \) several times slower than e.g. Marconi).
- Relatively large overhead:
  - Grid in \(\alpha\) and \(\lambda\): 0.2 s.
  - Bounce integrals 0.9 s.
  - Integral of \(g\) in \(\lambda\): 0.2 s.
- Easy to parallelize: \(v, \psi,\) bounce integrals, \(\partial_\psi \varphi_0, \varphi_1\) (next slide)... 

* One could think that anything below (say) 1 s is equally fine, but note that solving ambipolarity, quasineutrality, transport balance... requires evaluating the drift kinetic equation MANY times.
Solving quasineutrality (matrix response approach)

\[
\left[I_{vM,\alpha}(\alpha, \lambda) + \frac{1}{v_d} I_{vE,\alpha}(\alpha, \lambda)\right] \partial_\alpha g(\alpha, \lambda) - \frac{v\nu}{v_d} \partial_\lambda I_\nu(\alpha, \lambda) \partial_\lambda g(\alpha, \lambda) = \\
= \left( I_{vM,\psi}(\alpha, \lambda) - \frac{1}{v_d} \int_{l_{b1}}^{l_{b2}} \frac{dl}{\sqrt{1 - \lambda B}} \partial_\theta \varphi_1 \right) F_M \Upsilon, \quad (1)
\]

\[
\left( \frac{Z}{T} + \frac{1}{T_e} \right) \varphi_1 = \frac{2\pi}{en} \int_0^\infty dv \int_{B_{\text{max}}^{-1}}^{B^{-1}} d\lambda \frac{v^3 B}{|v_\parallel|} g. \quad (2)
\]

If we parametrize \( \varphi_1 \) using \( N \) vectors \( u_k \) and \( N \) coefficients, then the linear system can be esquematically written as

\[
\varphi_1 = \varphi_1^0 + A \varphi_1 \quad (3)
\]

where \( A \) is a \( N \times N \) matrix:

- \( \varphi_1^0 \) obtained by solving (1) with \( \varphi_1 = 0 \) and then using (2).
- The \( kth \) row of \( A \) is obtained by solving (1) with \( \varphi_1 = u_k \), solving (2) and substracting \( \varphi_1^0 \).
  - DKE is solved \( N + 1 \) times, but LU factorization is done once (e.g., in prev. case, for \( N = 1000 \), CPU time is 40 times larger).
- Once \( \varphi_1^0 \) and \( A \) are known, the linear system (3) can be solved.
Besides, depending on the plasma parameters, affecting $Q_i$, we know that $\varphi_1$ caused by the bulk species may have an important effect on radial transport of collisional impurities.

Yet, only a total of $\sim 100$ calculations of $\varphi_1$ (and of $\Gamma_Z$) have been published in papers [García-Regaña, NF (2013), García-Regaña, NF (2017), García-Regaña, NF (2018), Mollén, PPCF (2018), Velasco, PPCF (2018)].

Roughly speaking, this covers: $\sim 5$ magnetic configurations $\times$ 5 radial positions $\times$ 5 different plasma profiles.

More comprehensive study (dependence on the configuration, collisionality, bulk plasma profiles...) remains to be done.

We will do so by combining $\varphi_1$ calculated with KNOSOS and analytical formulas derived the flux of collisional impurities [Calvo, arXiv (2018), TTF (last week)].
Example (II): $\Gamma_{W44}$ vs. bulk profile shape

- Same plasma parameters as in [Calvo, arXiv (2018), TTF (last week)].
- LHD\(^6\) ($R_0 = 3.67$ m) at $\sqrt{\psi/\psi_{LCMS}} = 0.8$.
- $n = 1.2 \times 10^{20}$ m\(^3\), $T = 1.3$ keV.
- $E_r = \frac{T}{2e} \left( \frac{\partial_r n}{n} + 3.37 \frac{\partial_r T}{T} \right)$ (ion root limit, $1/\nu$ regime).

1008 calculations of $\varphi_1$ and $\Gamma_{W44}$.

For $\varphi_1 = 0$, outward pinch ($V > 0$) if $\partial_r T/T > 2\partial_r n/n$ [Helander, PRL (2017)].

With $\varphi_1 \neq 0$, outward pinch for hollow density only.

- Experimental evidence: [Huang, NF (2017)].

\(^6\) For this plot, we used a preliminary version of KNOSOS that makes certain assumptions on the magnetic configuration [Velasco, PPCF (2018)].
We have developed the code KNOSOS. It can provide very fast calculations of:

- Level of transport in several low-collisionality regimes ($1/\nu$, $\sqrt{\nu}$ and superbanana-plateau) for arbitrary geometry.
- DKES-like radial fluxes including the effect of $v_M \cdot \nabla \alpha$.
- $\varphi_1$ and $E_r$ (ongoing).

Benchmarking and experimental validation:

- Radial fluxes against DKES and FORTEC-3D.
- $\varphi_1$ against EUTERPE, SFINCS, FORTEC-3D and analytical expressions of [Calvo, JPP (2018)].
- Comparison with measurements (e.g. $Q_i$ and $E_r$ in W7-X [Alonso, EPS (2017), TTF (2018)]).

Next steps:

- Transition between $1/\nu$ and plateau.
- Solve DKE for electrons.
- Improve collision operator.
Energy transport simulations
Equations
Algorithms
First results
Ongoing work