Electron driven modes in fusion pedestals

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Outline

1. Overview of JET pedestal shots
2. Gyrokinetic results
3. Theory
Background

- Pedestal is an edge region of improved plasma performance: reduced turbulent fluctuations, and large plasma gradients.
- Differential rotation regularly cited as a reason for pedestal.
- What kind of microinstabilities do we have in this system?
- How much can we learn about the pedestal by studying its microinstabilities?
- In this talk, I will be emphasizing dynamics general to all pedestals we have studied.
- The promised land: predict pedestal height and width from 'first principles'.
Pedestal location

- Pedestal extends from the LCFS into the plasma, typically extends to $\psi_N = 0.95 - 0.90$.
- Pedestal region is highlighted in the figure of a toroidal cross section.
JET shots in this talk are largely controlled by fueling:

- **82550**
  - Very high D-gas injection, no seeded impurities, high triangularity, low $T_i$.

- **92167**
  - High D-gas injection.

- **92168**
  - Low D-gas injection.

- **92174**
  - C2D4 gas injection (maintaining very similar electrons s$^{-1}$ as 92167).
JET temperature, density profiles

- $\Delta T_e > \Delta T_i$ in steep gradient region.
- Densities vary substantially.

Figure: Profiles for 4 shots
JET gradient profiles

- $R/L_{Te} \gg 1 \Rightarrow \omega_\ast e \eta_e \gg k_\perp \cdot v_{Me}$. One would think that electron slab-like modes will be dominant.
- Important: $\eta_e \sim 4 - 10$, $\eta_i \sim 1 \Rightarrow$ slab ITG weak.
JET linear spectra

- Linear spectra vary strongly between pedestals and locations in specific pedestal. Challenging to compare like-for-like between pedestals.
- All modes drift in $\omega_e$ direction.

**Figure:** Electrostatic linear GS2 spectra for 4 JET shots at $\psi_N = 0.974$. 
Gradient sensitivities

Some weird behavior:

- Modes don’t seem to care much about $L_{Ti}$, but care about $L_{Te}$: below $\eta_e^{\text{critical}}$, growth strongly damped at wide range of scales.
  - $\eta_e \gg \eta_i \sim 1$ crucial for linear instability in all the pedestals we have studied.
- Reminiscent of ubiquitous mode (UM) (Coppi, 1977), but not quite the same. UM requires $\gamma \tau_b \ll 1$. Here, $\gamma \tau_b \gg 1 \Rightarrow$ trapped and passing electrons sample mode similarly!
Gradient sensitivities continued

Zooming in on ion-scales:

- Mode becomes slab once $R/L_{Ti} \approx 80 - 160$, $\eta_i \sim 10 - 20$.
- Compare $L_{Ti}$ to $L_s \equiv \hat{s}/qR$. Previous work (Newton, 2010) has shown that $L_s/L_{Ti} \approx 40$ before ITG instability threshold in sheared slab.

In this run, $\eta_e = 4$, $\eta_i = 1$. When $\eta_i \approx 5 - 10 > \eta_e$, electron mode dominates.

(c) Linear ion-scale spectra

(d) Eigenmodes for $k_y \rho_i = 0.40$
Gradient sensitivities continued

Zooming in on ion-scales:

- $R/L_{Te}$ threshold for ion scale $\Rightarrow$ electron gradients driving instability at ion scales.

![Linear ion-scale spectra](image)
JET Eigenmodes

Looking at eigenmodes, the plot thickens...

- In simulations we see a mix of slab-like modes, and modes extended along the field line, depending on the perpendicular mode size.
- Why non-slab modes at $k_y \rho_i \sim 1$ if $\omega_e \eta_e / \omega_{De} \gg 1$?
- Eigenmodes at $k_y \rho_i \sim 1$ do not resemble TEMs.
Non-slab modes

We have some clues on non-slab modes:

- The modes live at large $\theta$. Magnetic drift has term $-\theta k_\alpha \mathbf{v}_M \cdot \nabla q$. Let’s hack GS2 and remove the term.
- Slab versus non-slab cares about $k_y$ (but why?).

Set $\theta k_\alpha \mathbf{v}_M \cdot \nabla q = 0$, and modes become more slab-like, lose the $k_y \rho_i \sim 1 - 4$ peak:

(a) Linear spectra  (b) Eigenmodes with and without magnetic shear drift term for two different $k_y \rho_i$ values
Electron-driven modes?

- Evidence points to instability being driven primarily by electrons:
  - Real frequency of all modes in $\omega_{*e}$ direction.
  - Sensitivity to $\eta_e$ instead of $\eta_i$.
  - No instability with adiabatic electrons.
  - Modes, growth rates insensitive to $m_i$. 
Can we describe these electron slab-like modes analytically? Motivated by simulations and profiles, we use the orderings:

\[ \eta_e \gg \eta_i \sim 1, \quad \zeta_i \equiv \frac{\omega}{k_z v_{ti}} \gg 1, \quad \omega \sim \omega_{*e}, \quad k_y \rho_i \sim 1. \]

Begin with collisionless \( \beta = 0 \) unsheared slab gyrokinetic equation,

\[
\partial_t h_i + w || \hat{b} \cdot \nabla_R h_i = \frac{e f_{Mi}}{T_i} \partial_t \langle \phi_1^t \rangle_R \\
+ \frac{c}{B} (\nabla_R \langle \phi_1^t \rangle_R \times \hat{b}) \cdot \nabla x \left( \partial_x \ln n_e + \left( \frac{m_i \epsilon_0}{T_i} - \frac{3}{2} \right) \partial_x \ln T_i \right) f_{Mi},
\]

(1)

This gives the dispersion relation:

\[
1 + \frac{T_e}{T_i} \left( 1 - \Gamma_0 \right) - \frac{\omega_{*e}}{k_z v_{ti}} \left( \Gamma_0 \frac{1}{\zeta_i} + \eta_i b_i \frac{1}{\zeta_i} (\Gamma_1 - \Gamma_0) \right) \\
+ \frac{\omega_{*e}}{k_z v_{te}} \eta_e \left( \frac{Z(\zeta_e)}{2} - \zeta_e - \zeta_e^2 Z(\zeta_e) \right) + \frac{Z(\zeta_e)}{k_z v_{te}} \omega - \frac{Z(\zeta_e)}{k_z v_{te}} \omega_{*e} = 0.
\]

(2)
Slab theory gives surprising result.

Slab theory captures the ion scale bump, but simulations show the bump is due to magnetic drifts. We seem to be getting a reasonable answer for the wrong reason.
(In)Effectiveness of flow shear?

Flow shear appears ineffective at suppressing linear instabilities across a wide range of $k_y$ modes.

$$\tau_{floquet} = \frac{2\pi \hat{s}}{\gamma_{E}^{GS2}} \approx 50$$

$$\hat{s} = 6.2$$

Figure: Perturbed ion density versus time for a GS2 simulation with flow shear.
(In)Effectiveness of flow shear?

Another curious result:

- Relative invariance of $\gamma$ versus $\theta_0$ for low $k_y \rho_i \Rightarrow$ slab branch rather than toroidal branch.
- Seems counterintuitive: $\theta_0$ independent branches are also the least slab-like eigenmodes
- $\theta_0$ independent branches should be harder to shear: mode doesn’t care much about poloidal position.

![Figure](image)

**Figure:** Growth rates versus $\theta_0 = k_x/(2\pi k_y)$ for a range of $k_y$. 
Electromagnetic effects

Finite $\beta$ might be important at some scales, both for linear stabilization and drive.

Figure: $\beta$ scan: $\gamma$ versus $k_y$ for JET 82550 at $\psi_N = 0.984$. 

JET Shot82550 Linear run in the pedestal $Im(\omega)$, $\gamma_E = 0$, collisions, $\beta$ scan, $\psi = 0.984$
Conclusions

- Linear pedestal instabilities are electron-driven.
- Flow shear has not suppressed these microinstabilities in linear simulations.
- Instabilities in middle of pedestal are a mixture of slab and magnetic drift-driven modes
- $\Delta T_e > \Delta T_i \Rightarrow \eta_e > \eta_i$ crucial. If $\eta_e = \eta_i \approx 1$, very little instability. Most previous pedestal gyrokinetic simulations have used $T_e = T_i$. 
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Backup slides
$R_{LTe}$ versus $\Delta T_e$
JET Eigenmodes

Observations:

1. At $k_y \rho_i \gg 1$, slab-like modes dominate.
2. At $k_y \rho_i \sim 1$, we see fastest growing modes living at large $\theta$.

We can explain the large $\theta$ modes as magnetic drift driven modes. Consider the magnetic term in the GKE, $i \mathbf{v}_{Me} \cdot k_\perp h_e$:

$$\mathbf{v}_{Me} \cdot k_\perp = \mathbf{v}_{Me} \cdot \left( k_\psi \nabla \psi + k_\alpha \nabla \alpha \right)$$

$$= \mathbf{v}_{Me} \cdot \left( \nabla \psi (k_\psi + \theta \frac{dq}{d\psi} k_\alpha) + k_\alpha (\nabla \zeta + q \nabla \theta) \right)$$

(3)

The $k_\psi \theta \frac{dq}{d\psi}$ term is responsible for perturbations extended along the field line — can only get big at large $\theta$. What if we remove this term?
Figure: Comparison of fastest growing linear modes in GS2 pedestal shot 92174. Top: with all terms. Bottom: \( \mathbf{v}_M \cdot \theta \nabla q k_\alpha = 0 \).
The effects of the drifts on growth rates are manifest at a range of perpendicular scales: $v_M \cdot \theta \nabla q k_\alpha$ term removes ion-scale bump.
Theory

This dispersion relation describes the slab-branch of the dispersion relation fairly well when we ignore the magnetic drifts. The effects of the drifts on growth rates are manifest at many scales: $\mathbf{v}_M \cdot \theta \nabla q k_\alpha$ term removes ion-scale scale bump.

Shot 92174 $\psi_N = 0.974$ pedestal points
1. Solving the dispersion relation by comparing it with GS2 gyrokinetic simulation, we obtain reasonable agreement for $k_y \rho_i \sim 1$.

2. We have shown the fastest growing modes in the theory and from GS2.
Rotation and flow shear

Quick and dirty way to calculate flow shear: Motivated by previous works (Hatch et al. 2017 [Hatch2017]), we make the mean flow small,

\[ u_s = -\frac{\partial \tilde{\phi}}{\partial \psi} R^2 \nabla \zeta - \frac{\partial \tilde{p}_s}{\partial \psi} \frac{1}{Z_s e_n_s} R^2 \nabla \zeta + \frac{B}{n_s} K_s(\psi) \ll \frac{\rho_i}{L_{p_i}} v_{ti}. \]

\[ \equiv R\Omega_E \sim R \rho_{*,i} v_{ti} \]

\[ \equiv R\Omega_d \sim R \rho_{*,i} v_{ti} \]

\[ \equiv R\Omega_T \]

\[ \equiv \rho_{*,i} \]

(4)

**Approximation**: we neglect the purely parallel flow, obtaining \( \Omega_d \approx -\Omega_E \):

\[ -\frac{\partial \tilde{\phi}}{\partial \psi} \approx \frac{\partial \tilde{p}_s}{\partial \psi} \frac{1}{Z_s e_n_s}. \]

(5)

We then approximate \( \gamma_E(\psi) \), the shear in the mean rotation,

\[ \gamma_E \equiv \frac{\psi}{q} \frac{d\Omega_E}{d\psi} \approx -\frac{\psi}{q} \frac{d\Omega_d}{d\psi}. \]

(6)

We could include neoclassical flows in future work.
JET simulation locations

Gyrokinetic simulations chosen at 2 points for each simulation

1. The top of the pedestal where $R/L_n = 1.12$

2. The location where the shear is maximum where $d^3 p_i / d\psi^3 = 0$. 
JET linear spectra at $d^3 p_i/d\psi^3 = 0$

Figure: $\gamma$ versus $k_y$ for the four JET runs, $\beta = 0$.

- At $d^3 p_i/d\psi^3 = 0$, shots with wider and higher pedestals tend to have less unstable linear modes.
(In)Effectiveness of flow shear?

- Basic intuition for why it is ineffective, based on Newton et al. 2010 [Newton2010].
  - Eddies travel at speed along field line, $\dot{\theta} \approx \gamma_E/\hat{s}$.
  - If $\dot{\theta} > v_{ti}$, eddies move faster than thermal bulk of particles, and hence are sheared.
  - Eddies might be moving along field line too slowly to be sheared.
  - For slab-like modes, mode grows at same rate for all $\theta_0$, so movement to different poloidal position by flow doesn’t change growth rate.

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**Diagram:**

(a) $B = B_0(\hat{z} + \frac{x}{L_y} \hat{y})$

(b) $t = \frac{L_y}{V_0 \hat{e}_y \cdot \hat{y}}$
Changes in $\eta_s$

In max shear location, growth rates insensitive to ion gradients, very sensitive to electron gradients: minimal ITG in the pedestals we have examined:

**Figure:** Changes in growth rates by changing $L_{Ts}$. 
TEM-like modes?

Turning off trapped electron modes, the growth rates decrease, but the $\theta_0$ profile doesn’t change in shape.

(a) Growth rates versus $\theta_0$, tpdriftknob (b) Growth rates versus $\theta_0$, tpdriftknob = 1.

**Figure:** The effect of turning off trapped particle magnetic drifts.
JET linear spectra at pedestal top.

Figure: $\gamma$ versus $k_y$ for the four JET runs, $\beta = 0$.

- $k_y < 1$ ion scale modes are suppressed at the pedestal top for all shots, but driven strongly at $d^3 p_i / d\psi^3 = 0$ point.
- For $k_y > 1$, growth rates are much lower than at $d^3 p_i / d\psi^3 = 0$ point.
General Fourier Geometry

JET Shot #82550 $\psi = -0.02$
250 Fourier coefficients, 500 spatial points

$\epsilon = \frac{1}{N} \sum_i \sqrt{(Z_i - Z_{F,i})^2 + (R_i - R_{F,i})^2}$

Figure
Rotation and flow shear

So, how well does \( \Omega_E = -\Omega_d \) compare with the data?

\[
\Omega_d = -\frac{1}{Z_i e n_i d\psi} \\
\Omega_E = -\Omega_d \\
\Omega_1^{12}C^+ - \Omega_d = \Omega_E + \Omega_T
\]

**Figure:** JET pedestal rotation profile using radial force balance \( \nabla \rho_s = -Z_s e n_s \nabla \phi \) and neglecting the temperature gradient flow for \( u_s \).
$R/L_{Te}, R/L_{Ti}$

(a) $R/L_{Te}$

(b) $R/L_{Ti}$
Miller geometry

Miller geometry is a fairly good approximation for our flux surfaces.

Figure: Miller geometry fits for JET shot 92174. Top: \( \psi_N = 0.92 \), bottom \( \psi_N = 0.974 \).
$\eta_i, \eta_e$ are stability parameters in microinstability analysis; typical picture is if $\eta_i, \eta_e > \eta_{i,crit}, \eta_{e,crit}$, ion and electron temperature gradient instabilities ensue.

$$\eta_s \equiv \frac{L_{n,s}}{L_{T,s}}. \quad (7)$$
JET shear profiles

We need neoclassical corrections to calculate the correct flow shear, as our quick and dirty approach is very approximate:

\[
\psi
\]

Our shear profiles differ from the data, not surprising:

1. Figure ??: \( \gamma_E = -\frac{\psi}{q} \frac{d\Omega_d}{d\psi} \approx (\psi/q) \frac{d\Omega_E}{d\psi} \).

2. Figure ??: \( \gamma_{E_{\text{data}}} = \frac{\psi}{q} \frac{d}{d\psi}(\Omega_{\text{data}} - \Omega_d) = \frac{\psi}{q} \frac{d}{d\psi}(\Omega_E + \Omega_T) \).